Tranverse momentum broadening and the jet quenching parameter, Redux

based on: FDE, H. Liu and K. Rajagopal, to appear soon

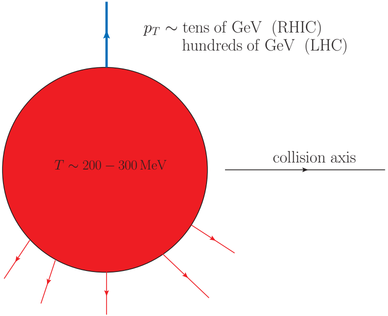
Francesco D'Eramo

Massachusetts Institute of Technology

Berkeley School of Collective Dynamics in High Energy Collisions

Outline

Motivation



High p_T hadrons suppression

Medium produced in the RHIC collisions is able to quench jets. Occasionally the hard valence partons undergo hard scattering, two back-to-back hard partons with a large p_T in the final state.

High p_T partons as probes

- the parton propagates as much as $5 - 10\,\mathrm{fm}$ within the medium
- production cross-sections for hard partons well known (both by pQCD and data from proton-nucleus collisions)

The medium has two main effects on the propagating hard parton:

- changing direction of its momentum
- parton energy loss

Transverse momentum broadening

Transverse momentum broadening

Change in momentum direction: “transverse momentum broadening”.

tranverse: perpendicular to the original direction of motion

broadening: many hard partons within a jet are kicked from the medium, no change in the mean momentum but the spread of the momenta of the individual partons broadens

The jet quenching parameter $\hat{q}$

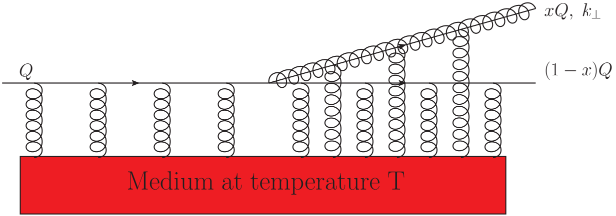
The jet quenching parameter is defined as the mean transverse momentum picked up by the hard parton per unit distance travelled (or in the high energy limit unit time)

$$\hat{q} = \frac{\langle k_\perp^2 \rangle}{L}, \qquad (L = \text{ medium length})$$

Energy loss in the high energy limit

Radiative energy loss

In the high energy limit enegy loss dominated by the QCD analogue of bremsstrahlung.



The incident and outgoing partons and the radiated gluon are constantly kicked by the medium: they are all subjects to transverse momentum broadening.
The jet-quenching parameter $\hat{q}$ plays a central role in the energy loss calculation, but it is defined via transverse momentum broadening only.

Toward a factorized description

Separation of scales

Parton energy loss and transverse momentum broadening involve widely separated scales

$$Q \gg k_\perp \gg T$$

Factorized description physics at each scale cleanly separated at lowest nontrivial order, correction to factorization systematically calculable, order by order in the small ratio between the scales.

First step

Formulation of the jet quenching parameter calculation in the language of Soft Collinear Effective Theory (SCET).

Set-up of the problem

Energy scales

Study propagation of a hard parton with initial four momentum

$q_0 \equiv (q_0^+, q_0^-, q_{0\perp}) = (0, Q, 0)$

propagating through some form of QCD matter. Consider QGP in equilibrium at temperature T (although our analysis would apply to other forms of matter).

We assume $Q \gg T$, we have a small dimensionless ratio $\lambda \equiv \frac{T}{Q} \ll 1$.

Goal

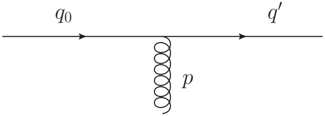
Characterize the transverse momentum broadening by computing $P(k_\perp)$, the probability distribution for the hard parton to acquire transverse momentum $k_\perp$ after traversing the medium.

$P(k_\perp)$ depends on the medium length L.

Normalization convention $\int \frac{d^2 k_\perp}{(2\pi)^2} P(k_\perp) = 1$

$k_\perp$ broadening in the high energy limit

$$q_0 = (0, Q, 0) \qquad q' = q_0 + p$$



Case 1: soft $p = (\lambda, \lambda, \lambda)Q$

Hard parton in the final state: $q' = Q(\lambda, 1, \lambda)$
Process suppressed by $\alpha_s(\sqrt{TQ})$
Kicked off-shell by $q'^2 \sim \lambda Q^2$, it then radiates gluons.

Case 2: Glauber $p = (\lambda^2, \lambda^2, \lambda)Q$

Final state "collinear" parton: $q' = Q(\lambda^2, 1, \lambda)$
Further Glaubers keep the part off-shell by the same order, $q'^2 \sim \lambda^2 Q^2$, not induced radiation
Interaction vertex: $\alpha_s(T)$

Case 3: collinear $p = (\lambda^2, 1, \lambda)Q$

Dominant contribution to parton energy loss.
Interaction vertex: $\alpha_s(T)$

$k_\perp$ broadening without radiation, $Q \to \infty$ limit

Relevance of Glauber gluons

Process 1, 2 and 3 all yield momentum broadening of order $\sim \lambda Q \sim T$.

- we neglect process 3, radiative processes
- we neglect process 1, suppressed by $\alpha_s(\sqrt{TQ})$
- process 2 larger contribution to momentum broadening in the $Q \to \infty$ limit
- Glauber gluons in the medium less numerous than the soft gluons, process 1 may be relevant at the Q values accessible at RHIC and the LHC.
  All 3 processes must be included before comparing to data.

Our focus

Non-radiative momentum broadening in the $Q \to \infty$ limit:

- easiest case to handle
- natural context in which the jet quenching parameter arises

Our language

Soft Collinear Effective Theory (SCET). In the $\lambda \to 0$ limit:

- natural separation of scales
- natural organization of the modes into kinematic regimes

SCET effective Lagrangian

Goal

Derive an effective Lagrangian to describe the interaction between collinear quarks and Glauber gluons. (Idilbi, Majumder)

Collinear quark field

Collinear quark: four-momentum scaling as $q = Q\left(\lambda^2, 1, \lambda\right)$.

Light-cone unit vectors: $\bar{n} \equiv \frac{1}{\sqrt{2}}(1, 0, 0, -1)\,, \qquad n \equiv \frac{1}{\sqrt{2}}(1, 0, 0, 1)$.

Collinear quark field decomposition

$$\xi(x) = \xi_{\bar{n}}(x) + \xi_n(x), \qquad \xi_{\bar{n}}(x) \equiv \frac{\not{\bar{n}}\not{n}}{2}\xi(x), \qquad \xi_n(x) \equiv \frac{\not{n}\not{\bar{n}}}{2}\xi(x)$$

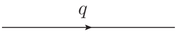
Integrating out the "small component" $\xi_n(x)$

Integrate out $\xi_n(x)$ by using its equations of motion

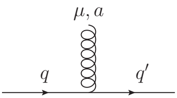
$$\mathcal{L}_{QCD} = \bar{\xi} i \not{D} \xi \quad \Rightarrow \quad \mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}}\, i\not{n}\,(\bar{n} \cdot D)\, \xi_{\bar{n}} + \bar{\xi}_{\bar{n}}\, i\not{D}_{\perp} \frac{1}{2\, in \cdot D}\, i\not{D}_{\perp}\, \not{n}\, \xi_{\bar{n}}$$

Effective Lagrangian at LO in λ and Feynman rules

- restrict to interactions with Glauber gluons in $D_\mu \equiv \partial_\mu - igA_\mu$, which can only change the perpendicular momentum $q_\perp$
- remove "large" phases from $\xi_{\bar{n}}(x)$: $\xi_{\bar{n}}(x) = e^{-iQx^+} \sum_{q_\perp} e^{iq_\perp \cdot x_\perp} \xi_{\bar{n},q_\perp}(x)$



$= i \,\bar{n}\!\!\!/ \frac{Q}{2q^+Q - q_\perp^2 + i\epsilon}$



$= igt^a n_\mu \,\bar{n}\!\!\!/$

Power counting in λ

$\xi_{\bar{n}}(x) \sim \lambda, \quad i\partial_\mu \xi_{\bar{n},q_\perp}(x) \sim \lambda^2 \xi_{\bar{n},q_\perp}(x), \quad A^+ \sim \lambda^2$

SCET Lagrangian at $\mathcal{O}(\lambda^4)$

$$\mathcal{L}_{\bar{n}} = \sum_{q_\perp, q'_\perp} e^{i(q_\perp - q'_\perp)\cdot x_\perp} \bar{\xi}_{\bar{n},q'_\perp} \left[i\bar{n}\cdot D + \frac{q_\perp^2}{2Q} \right] \not{n} \xi_{\bar{n},q_\perp}$$

Optical theorem

Unitarity of the S-matrix

Probability amplitude for the process $\alpha \to \beta$: $S_{\beta\alpha} = \delta_{\beta\alpha} + iM_{\beta\alpha}$.

The S-matrix is unitary: $\sum_\beta |S_{\beta\alpha}|^2 = 1 \Rightarrow 2\,\mathrm{Im}\; M_{\alpha\alpha} = \sum_\beta |M_{\beta\alpha}|^2$.

Box quantization

Cubic box of sides L. Periodic BC $\Rightarrow \mathbf{p} = \frac{2\pi}{L}(n_1, n_2, n_3)$.

In our set-up β differs from α only on its value of $k_\perp$: $\sum_\beta = L^2 \int \frac{d^2k_\perp}{(2\pi)^2}$.

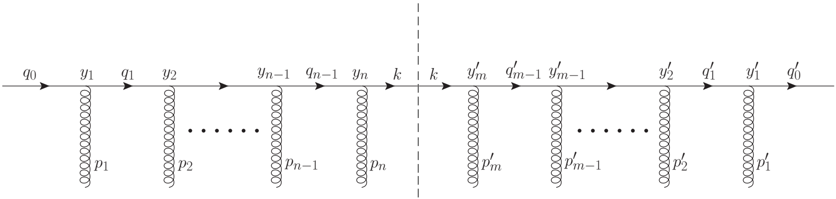
Probability distribution $P(k_\perp)$

We identify: $P(k_\perp) = L^2 \begin{cases} |M_{\beta\alpha}|^2 & \beta \neq \alpha \\ 1 - 2\mathrm{Im}\, M_{\alpha\alpha} + |M_{\alpha\alpha}|^2 & \beta = \alpha \end{cases}$

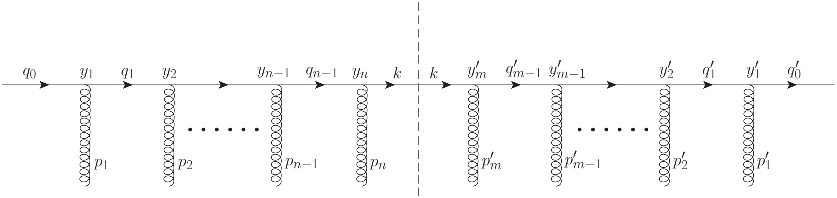
Forward scattering amplitude

Strategy

- Compute $2\,\mathrm{Im}\;M_{\alpha\alpha}$ by cutting the appropriate diagrams;
- Use the unitarity relation to identify $|M_{\beta\alpha}|^2$;
- Evaluate $P(k_\perp)$ for $k_\perp \neq 0$;
- The normalization condition $\int \frac{d^2k_\perp}{(2\pi)^2} P(k_\perp) = 1$ fixes $P(0)$.



$$2\,\mathrm{Im}\,M_{\alpha\alpha} = \sum_{m=1,n=1}^{\infty} \mathcal{A}_{mn} = \sum_{m=1,n=1}^{\infty} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2\mathcal{A}_{mn}}{d^2k_\perp}$$



SCET Lagrangian Feynman rules give:

$$
\begin{aligned}
\frac{d^2\mathcal{A}_{mn}}{d^2k_\perp} = & \frac{1}{\sqrt{2}QL^3}\int\frac{dk^+dk^-}{(2\pi)^2}\prod_{i=1}^{n-1}\frac{d^4q_i}{(2\pi)^4}\prod_{j=1}^{m-1}\frac{d^4q'_j}{(2\pi)^4} \\
& \times\bar{\xi}_{\bar{n}}(q'_0)\prod_{j=m-1}^{1}\left[(-ig)A^+(-p'_j)\,\not{n}\,\frac{-iQ}{2Qq'^{+}_j-q'^{\,2}_{j\perp}-i\epsilon}\,\bar{\not{n}}\right](-ig)A^+(-p'_m)\not{n} \\
& \times 2\pi Q\delta\left(2k^+Q-k_\perp^2\right)\bar{\not{n}}\;igA^+(p_n)\not{n}\prod_{i=1}^{n-1}\left[\frac{iQ}{2Qq^+_i-q^2_{i\perp}+i\epsilon}\,\bar{\not{n}}\,igA^+(p_i)\not{n}\right]\xi_{\bar{n}}(q_0)
\end{aligned}
$$

A few comments on $\frac{d^2\mathcal{A}_{mn}}{d^2k_\perp}$

The cut momentum $k_\perp$

The cut momentum k is the four-momentum of the hard parton in the final state. Its perpendicular component $k_\perp$ is not integrated over.
For forward scattering amplitude: $q_0 = q'_0 \;\Rightarrow\; k_\perp = \sum_{i=1}^{n} p_{i\perp} = \sum_{i=1}^{m} p'_{i\perp}$

- $p_{i\perp}$'s and $p'_{i\perp}$'s are of order $\lambda Q = T$ in magnitude;
- $k_\perp$ may turn out to be larger.

Typical value of $k_\perp^2$ is $\hat{q}L$, in particular $k_\perp^2$ grows with L.

$A_\mu(p)$ as a background field

Hard parton propagation in a given field configuration $A_\mu(p)$. Nonperturbative physics of the medium does not enter this calculation. Average over configurations at the end.

Gluon operators $A^+ = A^{a+}\, t_F^a$ ordering

$\prod_{j=m-1}^{1} A^+(-p'_j) \equiv A^+(-p'_1)\cdots A^+(-p'_{m-1})$; $\prod_{i=1}^{n-1} A^+(p_i) \equiv A^+(p_{n-1})\cdots A^+(p_1)$

Forward scattering amplitude evaluation II

After

- averaging over the color indices;
- some Dirac algebra;
- gluon fields in the coordinate space

$$\frac{d^2\mathcal{A}_{mn}}{d^2k_\perp} = \frac{2^{n+m}}{\sqrt{2}L^3\, N_c} \int \prod_{i=1}^{n} d^4y_i \prod_{j=1}^{m} d^4y'_j \; e^{-iq_0\cdot(y_1-y'_1)} \; \mathrm{Tr}\left[\prod_{j=m}^{1}(-ig)A^+(y'_j) \prod_{i=1}^{n} igA^+(y_i)\right]$$
$$\times\, g(y_n - y'_m, k_\perp) \prod_{j=1}^{m-1} f^*(y'_j - y'_{j+1}) \prod_{i=1}^{n-1} f(y_i - y_{i+1})$$

$$f(z) \equiv \int \frac{d^4q}{(2\pi)^4} \frac{iQ}{2Qq^+ - q_\perp^2 + i\epsilon} e^{iq\cdot z} = \delta(z^+)\theta(-z^-)\frac{iQ}{4\pi z^-} e^{-i\frac{Q}{2z^-}z_\perp^2} ,$$

$$g(z, k_\perp) \equiv \int \frac{dk^+dk^-}{(2\pi)^2} 2\pi Q\delta\left(2k^+Q - k_\perp^2\right) e^{ik\cdot z} = \frac{1}{2}\delta(z^+)e^{-ik_\perp\cdot z_\perp + i\frac{k_\perp^2}{2Q}z^-} .$$

The $Q \to \infty$ limit

$Q \to \infty$ for $f(z)$ and $g(z, k_\perp)$

So far not used the $Q \to \infty$ limit (although used in setting up the problem). In this limit both $f(z)$ and $g(z, k_\perp)$ simplify.

- $Q \gg p_\perp^2 z^- \quad \Rightarrow \quad f(z) \approx \frac{1}{2}\delta(z^+)\,\theta(-z^-)\,\delta^2(z_\perp)$
- $Q \gg k_\perp^2 z^- \quad \Rightarrow \quad g(z, k_\perp) \approx \frac{1}{2}\delta(z^+)\, e^{-ik_\perp \cdot z_\perp}$

Condition for the $Q \to \infty$ limit

- $p_\perp^2 \sim T^2$ is the typical magnitude of the $p_{i\perp}$'s and $p'_{i\perp}$'s
- $k_\perp^2 = \left(\sum_{i=1}^n p_{i\perp}\right)^2 = \left(\sum_{i=1}^m p'_{i\perp}\right)^2$

Criteria for $g(z, k_\perp)$ stronger, z^- cannot be bigger than $L^- \equiv \sqrt{2}L$.
We require: $Q \gg k_\perp^2 L \sim \hat{q} L^2$

Physical significance of the $Q \to \infty$

If the condition $Q \gg k_\perp^2 L \sim \hat{q} L^2$ is satisfied

- $f(z) \propto \delta^2(z_\perp)$

It means that L is short enough that trajectory of the hard parton in position space remains well-approximated as a straight line, even though it picks up transverse momentum.

In this limit:

$$\sum_{m=1,n=1}^{\infty} \frac{d^2 \mathcal{A}_{nm}}{d^2 k_\perp} = \frac{1}{N_c} \int d^2 x_\perp \, e^{-ik_\perp \cdot x_\perp} \left\langle \mathrm{Tr}\left[\left(W_F^\dagger[0, x_\perp] - 1\right) \left(W_F[0,0] - 1\right)\right]\right\rangle$$

where

$$W_F\left[y^+, y_\perp\right] \equiv P\left\{\exp\left[ig \int_0^{L^-} dy^- \, A^+(y^+, y^-, y_\perp)\right]\right\}$$

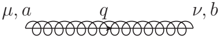
$|M_{\beta\alpha}|^2$ from the unitarity relation

The unitarity relation

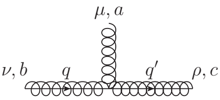
$$\int \frac{d^2 k_\perp}{(2\pi)^2} \sum_{n=1,m=1}^{\infty} \frac{d^2 \mathcal{A}_{nm}}{d^2 k_\perp} = 2\,\mathrm{Im}\; M_{\alpha\alpha} = \sum_{\beta} |M_{\beta\alpha}|^2$$

allows us to identify

$$|M_{\beta\alpha}|^2 = \frac{1}{L^2 N_c} \int d^2 x_\perp \, e^{-ik_\perp \cdot x_\perp} \left\langle \mathrm{Tr}\left[\left(W_F^\dagger[0, x_\perp] - 1 \right) \left(W_F[0,0] - 1 \right) \right] \right\rangle$$



$$= -i g_{\mu\nu} \frac{1}{2q^+ Q - q_\perp^2 + i\epsilon} \delta^{ab}$$



$$= -2ig\, q^- \, (t^a_G)_{bc} \, \bar{n}_\mu g_{\nu\rho}$$

Collinear gluon case

The propagation of a collinear gluon is analogous, but:

- different Feynman rules
- Glauber gluons in the adjoint

Probability distribution $P(k_\perp)$ and $\hat{q}$

Expression for $P(k_\perp)$

$|M_{\beta\alpha}|^2$ and $2\,\mathrm{Im}\,M_{\alpha\alpha}$ are all we need to find $P(k_\perp)$. Thus

$$P(k_\perp) = \int d^2x_\perp\, e^{ik_\perp\cdot x_\perp}\, \mathcal{W}_{\mathcal{R}}(x_\perp), \qquad \mathcal{W}_{\mathcal{R}}(x_\perp) = \frac{1}{d\,(\mathcal{R})}\left\langle \mathrm{Tr}\left[W^\dagger_{\mathcal{R}}[0, x_\perp]\, W_{\mathcal{R}}[0,0]\right]\right\rangle$$

for a collinear particle in the $SU(N)$ representation $\mathcal{R}$, with dimension $d\,(\mathcal{R})$.

Properties of $P(k_\perp)$

- $P(k_\perp)$ depends only on the medium property (thus also $\hat{q}$ does).
- Transverse momentum broadening without radiation: field theoretically well-defined property of the medium.
- This is the kind of factorization we hope to find once radiation is included.

$\hat{q}$ from light-like Wilson lines

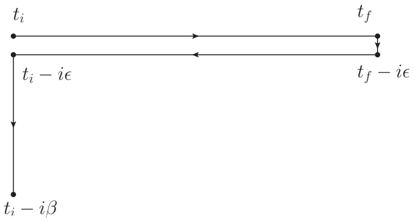
$$\hat{q} \equiv \frac{\langle k_\perp^2\rangle}{L} = \frac{\sqrt{2}}{L^-}\int \frac{d^2k_\perp}{(2\pi)^2}\, k_\perp^2 \int d^2x_\perp\, e^{ik_\perp\cdot x_\perp}\,\mathcal{W}_{\mathcal{R}}(x_\perp)$$

Operator ordering for the $\hat{q}$ evaluation

Expectation value $\mathcal{W}_{\mathcal{R}}(x_\perp)$ different operator ordering than a standard Wilson loop. Recall $A^+ = (A^+)^a t^a$.

Standard Wilson loop

- $(A^+)^a$ time ordered;
- t^a path ordered.



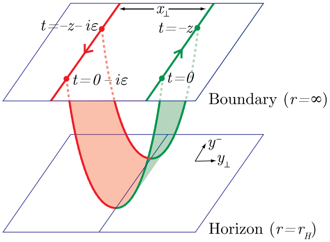
Wilson lines in $\mathcal{W}_{\mathcal{R}}(x_\perp)$

- $(A^+)^a$ path ordered;
- t^a path ordered.

$\mathcal{W}_{\mathcal{R}}(x_\perp)$ should be described using the Schwinger-Keldysh contour

- one of the light-like Wilson lines on the $\mathrm{Im}\, t = 0$ segment
- the other light-like Wilson line on the $\mathrm{Im}\, t = -i\epsilon$ segment

$\hat{q}$ evaluation in $\mathcal{N} = 4$ SYM revisited



Standard AdS/CFT evaluation

- $N = 4$ $SU(N_c)$ gauge theory
- large N_c and $g^2_{YM} N_c$ limit
- Gravity dual: AdS Schwarzschild black hole at nonzero temperature
- $\langle W(\mathcal{C}) \rangle = \exp\left[i \left\{ S(\mathcal{C}) - S_0 \right\} \right]$

(Liu, Rajagopal, Wiedemann)

Taking the ordering into account

(Lorentztian AdS/CFT, Skenderis and Van Rees)

- Construct the bulk geometry corresponding to the $\mathrm{Im}\, t = -i\epsilon$ segment of the Schwinger-Keldysh contour
- Two segments meet only at the horizon, only one nontrivial (connected) string world sheet
- String action solution at the boundary $r = \infty$, now not allowed: Wilson lines at different values of $\mathrm{Im}\, t$, no string world sheets that connect them without touching the horizon.
- same world sheet as the old calculation, result unchanged: $\hat{q} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})} \sqrt{g^2_{YM} N_c}\, T^3$

Summary and future directions

Summary

- Probability distribution $P(k_\perp)$ evaluation within an EFT formalism
- Glaubers responsible for $k_\perp$ broadening in the absence of radiation
- $P(k_\perp)$ and $\hat{q}$ depend on the medium property only (factorization)
- Subtleties about the operators ordering: strong coupling $\hat{q}$ evaluation more straightforward, previous result unchanged

Future directions

- include soft gluons (α_s suppressed, but more numerous)
- include radiation, see how $\hat{q}$ enters in the spectrum of the radiated gluons
- include higher order corrections in λ
- weak-coupling $\hat{q}$ evaluation for QCD plasma at high enough T
- compare our $P(k_\perp)$ with the correspondent quantity in $N = 4$ SYM